# Proof of the Free Space Condition

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## 1 Proof of the free space condition

$$\omega^a_b = k\epsilon^a{}_{bc}\, q^c$$

This fundamental condition is a solution of:

$$\boxed{R^a{}_b \wedge q^b = \omega^a{}_b \wedge T^b} \tag{1}$$

$$\left(D \wedge \omega^a{}_b\right) \wedge q^b = \omega^a{}_b \wedge \left(D \wedge q^b\right) \tag{2}$$

$$\left(d \wedge \omega^a{}_b\right) \wedge q^b + \left(\omega^a{}_c \wedge \omega^c{}_b\right) \wedge q^b = \omega^a{}_b \wedge \left(d \wedge q^b\right) + \omega^a{}_b \wedge \left(\omega^b{}_c \wedge q^c\right) \tag{3}$$

To Prove:

$$\left(d \wedge \omega^a{}_b\right) \wedge q^b = \omega^a{}_b \wedge \left(d \wedge q^b\right) \tag{4}$$

Proof

$$\left(d \wedge \omega^1{}_2\right) \wedge q^2 + \left(d \wedge \omega^1{}_3\right) \wedge q^3 = \omega^1{}_2 \wedge \left(d \wedge q^2\right) + \omega^1{}_3 \wedge \left(d \wedge q^3\right) \tag{5}$$

Eqn. (5) is true if

$$\omega^1{}_2 = k\epsilon^1{}_{23}\, q^3 = kq^3 \tag{6}$$

$$\omega^1{}_3 = k\epsilon^1{}_{32}\, q^2 = -kq^2 \tag{7}$$

i.e.

$$\left(d \wedge q^3\right) \wedge q^2 - \left(d \wedge q^2\right) \wedge q^3 = q^3 \wedge \left(d \wedge q^2\right) - q^2 \wedge \left(d \wedge q^3\right) \tag{8}$$

$$\begin{aligned} \Longrightarrow \quad & \left(d \wedge q^3\right) \wedge q^2 = -q^2 \wedge \left(d \wedge q^3\right) \\ & -\left(d \wedge q^2\right) \wedge q^3 = q^3 \wedge \left(d \wedge q^2\right) \end{aligned} \tag{9}$$

To prove

$$\left(\omega^a{}_c \wedge \omega^c{}_b\right) \wedge q^b = \omega^a{}_b \wedge \left(\omega^b{}_c \wedge q^c\right) \tag{10}$$

Proof

For $a$=1, $b$=2, $c$=3;

$$\left(\omega^1{}_3 \wedge \omega^3{}_2\right) \wedge q^2 = \omega^1{}_2 \wedge \left(\omega^2{}_3 \wedge q^3\right) \tag{11}$$

where

$$\omega^1{}_2 = kq^3; \quad \omega^1{}_3 = -kq^2$$
$$\omega^3{}_2 = -kq^1; \quad \omega^2{}_3 = kq^1$$

therefore

$$\left(q^2 \wedge q^1\right) \wedge q^2 = -q^3 \wedge \left(q^1 \wedge q^3\right)$$

i.e.

$$q^3 \wedge q^2 = -q^3 \wedge \left(-q^2\right)$$
$$= q^3 \wedge q^2 \tag{12}$$

For O(3) electrodynamics we choose:

$$\omega^a_b = -\frac{1}{2}\, k\epsilon^a{}_{bc}\, q^c \tag{13}$$

In the structure relation:

$$D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{14}$$

Proof

For $a$=1:

$$D \wedge q^1 = d \wedge q^1 - \frac{1}{2}\left(\epsilon^1{}_{23}\, q^3 \wedge q^2 + \epsilon^1{}_{32}\, q^2 \wedge q^3\right) \tag{15}$$

$$\boxed{D \wedge q^1 = d \wedge q^1 + kq^2 \wedge q^3}$$

In the O(3) circular complex basis this gives O(3) electrodynamics

This allows the tetrad of the free field to be identified as the potential, and also the spin connection. O(3) electrodynamics is therefore a fundamental theory of general relativity.