## NEW GENERAL CONDITION FOR ANY METRIC

Start with the definition of the tetrad:

$$
\begin{equation*}
V^{a}=q_{\mu}^{a} V^{\mu} \tag{1}
\end{equation*}
$$

A particular case of this is:

$$
\begin{equation*}
x^{\kappa}=g_{\mu}^{\kappa} x^{\mu} \tag{2}
\end{equation*}
$$

where $g_{\mu}^{\kappa}$ is the metric.

Consider:

$$
\begin{equation*}
D_{\mu} V^{v}=\partial_{\mu} V^{v}+\Gamma_{\mu \lambda}^{v} V^{\lambda} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
D_{\mu} V^{a}=\partial_{\mu} V^{a}+\omega_{\mu b}^{a} V^{b} \tag{4}
\end{equation*}
$$

Eqs. (3) and (4) imply the tetrad postulate:
i.e.

$$
\begin{gather*}
\partial_{\mu} q_{v}^{a}+\omega_{\mu b}^{a} q_{v}^{b}-\Gamma_{\mu \nu}^{\lambda} q_{\lambda}^{a}=0  \tag{5}\\
\Gamma_{\mu \nu}^{\lambda}=q_{a}^{\lambda}\left(\partial_{\mu} q_{v}^{a}+\omega_{\mu b}^{a} q_{v}^{b}\right) \tag{6}
\end{gather*}
$$

using

$$
\begin{equation*}
q_{\lambda}^{a} q_{a}^{\alpha}=\delta_{\lambda}^{\alpha} \tag{7}
\end{equation*}
$$

The special case of Eq.(2) implies that Eq. (4) becomes:

$$
\begin{equation*}
\partial_{\mu} g_{v}^{\kappa}+\Gamma_{\mu \lambda}^{\kappa} g_{\nu}^{\lambda}-\Gamma_{\mu \nu}^{\lambda} g_{\lambda}^{\kappa}=0 \tag{8}
\end{equation*}
$$

i.e. $a$ is replaced by $\kappa$, b by $\lambda$, and $\omega$ by $\Gamma$. In Eq. (8):

$$
\begin{equation*}
\Gamma_{\mu \lambda}^{K} g_{v}^{\lambda}=\Gamma_{\mu \nu}^{K} \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
\Gamma_{\mu \nu}^{\lambda} g_{\lambda}^{\kappa}=\Gamma_{\mu \nu}^{\kappa} \tag{10}
\end{equation*}
$$

Therefore:

$$
\begin{equation*}
\partial_{\mu} g_{v}^{\kappa}=0 \tag{11}
\end{equation*}
$$

This is an important new fundamental equation for the metric:

$$
\begin{equation*}
g_{v}^{K}=g^{\kappa \alpha} g_{\alpha v} \tag{12}
\end{equation*}
$$

Any Riemannian metric obeys Eq. (11), and in general any metric in any spacetime of any dimension, in general a spacetime with torsion and curvature.

Diagonal metric
In this case, off-diagonals are zero, so Eq. (11) produces:

$$
\begin{align*}
& \partial_{\mu}\left(g^{00} g_{00}\right)=0  \tag{13}\\
& \partial_{\mu}\left(g^{11} g_{11}\right)=0  \tag{14}\\
& \partial_{\mu}\left(g^{22} g_{22}\right)=0  \tag{15}\\
& \partial_{\mu}\left(g^{33} g_{33}\right)=0 \tag{16}
\end{align*}
$$

for any $\mu$. Eqs (13) - (16) are true because

$$
\begin{equation*}
g^{00} g_{00}=g^{11} g_{11}=g^{22} g_{22}=g^{33} g_{33}=1 \tag{17}
\end{equation*}
$$

In general, in four dimensions:

$$
\begin{equation*}
\partial_{\mu}\left(g^{\kappa \alpha} g_{\alpha v}\right)=0 \tag{18}
\end{equation*}
$$

where:

$$
\begin{equation*}
g^{\kappa \alpha} g_{\alpha v}=g^{\kappa 0} g_{0 v}+g^{\kappa 1} g_{1 v}+g^{\kappa 2} g_{2 v}+g^{\kappa 3} g_{3 v} \tag{19}
\end{equation*}
$$

and where the metric has diagonal and off-diagonal elements.

The metric of the orbital theorem of Paper 111 obeys Eq. (11) because it is a diagonal metric.

## Computer Test.

It is possible now to test metrics with off-diagonal elements by using computer algebra with Eq. (18).

