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SOME NOTES ON METRIC COMPATIBILITY  
AND THE TETRAD POSTULATE.

The metric compatibility condition of Riemann geometry is well known to be:

$$\partial_\rho g_{\mu\nu} = 0 \quad - (1)$$

where  $\partial_\rho$  is the covariant derivative and  $g_{\mu\nu}$  is symmetric metric. If it is assumed that the torsion tensor is zero:

$$T^k_{\mu\nu} = \Gamma^k_{\mu\nu} - \Gamma^k_{\nu\mu} = 0 \quad - (2)$$

we obtain the well known expression for the Christoffel connection of eqn (2):

$$\Gamma^\sigma_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} \left( \partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu} \right) \quad - (3)$$

Einstein and Hilbert used eqns (1), (2) and (3) to develop the famous theory of general relativity (1915 - 1916).

In the Palatini variation of general relativity, developed mainly by Cartan, the tetrad  $\eta^\alpha_\mu$  is the fundamental field.

2) The symmetric metric is:

$$g_{\mu\nu} = g_{\mu}^a g_{\nu}^b \eta_{ab} \quad - (4)$$

where  $\eta_{ab}$  is the Minkowski metric of the tangent bundle. The tetrad postulate is:

$$\boxed{D_{\sigma} g_{\mu}^a = 0} \quad - (5)$$

and is true for any connection. The tetrad postulate is not predicated on the assumption of metric compatibility or absence of torsion.

The metric compatibility of Riemann geometry is obtained from the tetrad postulate as follows. From eqn. (4):

$$D_{\sigma} g_{\mu\nu} = D_{\sigma} (g_{\mu}^a g_{\nu}^b \eta_{ab}) \quad - (6)$$

$$= \eta_{ab} (g_{\nu}^b D_{\sigma} g_{\mu}^a + g_{\mu}^a D_{\sigma} g_{\nu}^b)$$

$$= 0$$

where we have used eqn (5) and the Leibniz theorem, together with:  $D_{\sigma} \eta_{ab} = 0$ . - (7)

Therefore given eqn. (4) we obtain eqn. (1) for eqn. (5). This is a powerful and simple demonstration that the tetrad postulate is more fundamental than the compatibility condition used by Einstein and Hilbert.