

374(4) : Time Dependent \propto Factor and Conservation of Angular Momentum.

From UFT 363 the momentum is:

$$\underline{p} = m \underline{v} = m (\alpha \dot{r} \underline{e}_r + r \dot{\phi} \underline{e}_\phi) \quad - (1)$$

also $\alpha = 1 + \frac{dR_r}{dr} \quad - (2)$

and $R_r = R_r(r(t), \phi(t), t) \quad - (3)$

So in general R_r depends on time, so α also depends on time. The force equation is:

$$\underline{F} = \underline{\dot{p}} = m \frac{d}{dt} (\alpha \dot{r} \underline{e}_r + r \dot{\phi} \underline{e}_\phi) = -\frac{mMG}{r^2} \underline{e}_r \quad - (4)$$

It follows that:

$$\alpha \ddot{r} \underline{e}_r + \alpha \frac{d}{dt} (\dot{r} \underline{e}_r) + \frac{d}{dt} (r \dot{\phi} \underline{e}_\phi) = -\frac{mG}{r^2} \underline{e}_r \quad - (5)$$

i.e. $(\dot{r} \alpha + \alpha \ddot{r} - r \dot{\phi}^2) \underline{e}_r + ((\alpha + 1) \dot{r} \dot{\phi} + r \ddot{\phi}) \underline{e}_\phi = -\frac{mG}{r^2} \underline{e}_r \quad - (6)$

It follows that:

$$\alpha \ddot{r} + \alpha \dot{r} - \dot{\phi}^2 r = -\frac{mG}{r^2} \quad - (7)$$

$$(\alpha + 1) \dot{r} \dot{\phi} + r \ddot{\phi} = 0 \quad - (8)$$

$$\dot{\phi} = \frac{L}{mr^2} \quad - (9)$$

2) This system of equations can be solved for the orbital function:

$$\frac{d\phi}{dr} = \frac{\dot{\phi}}{\dot{r}} \quad - (10)$$

in terms of x and \dot{x} , where:

$$\dot{x} = \frac{d}{dt} \left(\frac{\partial R_r}{\partial r} \right) \quad - (11)$$

and

$$x = 1 + \frac{\partial R_r}{\partial r} \quad - (12)$$

The orbit is:

$$r = \int \frac{dr}{d\phi} d\phi \quad - (13)$$

and in previous work it was found that it is a precessing orbit.

Fluid gravitation gives a precessing planar orbit.

If it is assumed that spacetime is an inviscid, incompressible fluid:

$$\underline{\nabla} \cdot \underline{v} = 0 \quad - (14)$$

and this assumption gives another equation as described in note 374(1):

$$\frac{1}{r} \left(x\dot{r} + \dot{\phi} \frac{\partial r}{\partial \phi} \right) + \frac{\partial}{\partial r} (x\dot{r}) + \frac{\partial \dot{\phi}}{\partial \phi} = 0 \quad - (15)$$

Eq. (15) can be solved simultaneously w/4 eqns. (7) to (9).

3) Therefore a spacetime of type (14) give a relation between x and $dr/d\phi$ in eq. (15).

The vorticity can be defined by:

$$\underline{w} = \underline{\nabla} \times \underline{v} \quad (16)$$

where

$$\underline{v} = x \dot{r} \underline{e}_r + r \dot{\phi} \underline{e}_\phi \quad (17)$$

and x defined by eqns. (2) and (3). Moreover, \underline{w} is a vorticity field of spacetime:

$$\underline{w} = \underline{w}(r(t), \phi(t), t) \quad (18)$$

Conservation of angular momentum means that:

$$\frac{d\underline{w}}{dt} + \underline{\nabla} \times (\underline{w} \times \underline{v}) = \frac{1}{R} \nabla^2 \underline{w} \quad (19)$$

where R is a spacetime Reynolds number.

Therefore eqs. (7) to (9) and (19) could be solved simultaneously for the orbit (13) and for x and \dot{x} , possibly giving a turbulent orbit.